

Anomalous magnetic moment of the muon in the two Higgs doublet model

E. O. Iltan *

Physics Department, Middle East Technical University
Ankara, Turkey

H. Sundu †

Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We calculate the new physics effects on the anomalous magnetic moment of the muon in the framework of the two Higgs doublet model. We predict an upper bound for the lepton flavor violating coupling, which is responsible for the point like interaction between muon and tau, by using the uncertainty in the experimental result of the muon anomalous magnetic moment. We show that the upper bound predicted is more stringent compared to the one which is obtained by using the experimental result of the muon electric dipole moment.

*E-mail address: eiltan@heraklit.physics.metu.edu.tr

†E-mail address: sundu@metu.edu.tr

1 Introduction

The lepton flavor violating (LFV) interactions, non-zero electric dipole moments (EDM) and the anomalous magnetic moments (AMM) of leptons are among the most promising candidates to search for physics beyond the standard model (SM). The AMM of the muon have been studied in the literature extensively [1] and [2]. The experimental result of the muon AMM by the g-2 Collaboration [3] has been obtained as

$$a_\mu = 116\,592\,023(151) \times 10^{-11} , \quad (1)$$

and recently, at BNL [4], a new experimental world average has been announced

$$a_\mu = 11\,659\,203(8) \times 10^{-10} , \quad (2)$$

which has about half of the uncertainty of previous measurements. This result has opened a new window for testing the SM and the new physics effects beyond. The SM prediction for a_μ is written in terms of different contributions [5];

$$a_\mu(SM) = a_\mu(QED) + a_\mu(weak) + a_\mu(hadronic) , \quad (3)$$

where $a_\mu(QED) = 11\,658\,470.57(0.29) \times 10^{-10}$ and $a_\mu(weak) = 15.1(0.4) \times 10^{-10}$. The hadronic contributions are under theoretical investigation. With the new data from Novosibirsk [6], the calculation of the first order hadronic vacuum polarization to $a_\mu(SM)$ is obtained as $684.7(7.0) \times 10^{-10}$ ($701.9(6.1) \times 10^{-10}$) using the e^+e^- (τ) based result. The addition of the higher order contributions, $-10.0(0.6) \times 10^{-10}$ and light by light scattering $-8.6(3.2) \times 10^{-10}$, result in $a_\mu(SM) = 11\,659\,169.1(7.8) \times 10^{-10}$ ($a_\mu(SM) = 11\,659\,186.3(7.1) \times 10^{-10}$ based on e^+e^- (τ) data. Therefore, there is a 3.0 (1.6) standard deviation from the experimental result and this could possibly be due to the effects of new physics, at present.

Various scenarios have been proposed to explain the nonvanishing value of the deviation Δa_μ [7] - [24], previously. The Supersymmetry (SUSY) contribution to a_μ has been investigated in [2, 9, 14]. In [15] the new physics effect on a_μ has been explained by introducing a new light gauge boson. The prediction of the muon AMM has been estimated in the framework of leptoquark models in [16], the technicolor model with scalars and top color assisted technicolor model in [19], in the framework of the general two Higgs doublet model (2HDM) in [20] and also in [21]. The work [22] was devoted to the Higgs mediated lepton flavor violating interactions which contributed to a_μ . In this study, only the scalar Higgs exchange was taken into account by assuming that the pseudoscalar Higgs particle was sufficiently heavier than the

scalar one. Finally, in [24], scalar scenarios contributing to a_μ with enhanced Yukawa coupling were proposed.

In [1], the upper bound on leptonic flavor changing coupling, related with the transition $\tau-\mu$, has been obtained in the 2HDM as 0.11, using the AMM of the muon by considering that the dominant contribution comes from the lighter scalar boson. In this case the uncertainty between the SM prediction and the experimental one was taken as 7.4×10^{-9} and it was emphasized that this bound would decrease to the values of ~ 0.03 with the reduction of the uncertainty up to a factor 20.

In our work, we study the new physics effects on the AMM of the muon using the model III version of the 2HDM of reference [1], including both scalar and pseudoscalar Higgs boson effects, based on the assumption that the numerical value should not exceed the present experimental uncertainty, $\sim (1-2) \times 10^{-9}$. The new contribution to a_μ exists at one-loop level with internal mediating neutral particles h^0 and A^0 in our case, since we do not include charged FC interaction in the leptonic sector due to the small couplings for $\mu-\nu_l$ interactions. In the calculations, we take into account the internal τ and μ leptons and neglect the contribution coming from the internal e -lepton since the corresponding Yukawa coupling is expected to be smaller compared to the others. Furthermore, we also neglect the internal μ -lepton contribution by observing the weak dependence of Δa_μ on the μ - μ coupling. We predict a stringent upper bound for the μ - τ coupling and compare with the one, which is obtained by using the restriction coming from the EDM of μ lepton (see [25] for details).

The paper is organized as follows: In Section 2, we present the new physics effects on the AMM of the muon in the framework of the general 2HDM. Section 3 is devoted to discussion and our conclusions.

2 Anomalous magnetic moment of the muon in the model III version of two Higgs doublet model.

In the type III 2HDM, there exist flavor changing neutral currents (FCNC), mediated by the new Higgs bosons, at tree level. The most general Higgs-fermion interaction for the leptonic sector in this model reads as

$$\mathcal{L}_Y = \eta_{ij}^E \bar{l}_{iL} \phi_1 E_{jR} + \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. \quad , \quad (4)$$

where i, j are family indices of leptons, L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, l_{iL} and E_{jR} are lepton doublets and singlets respectively, ϕ_i for $i = 1, 2$, are the two scalar

doublets

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] ; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \quad (5)$$

with the vacuum expectation values

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; \langle \phi_2 \rangle = 0 . \quad (6)$$

With the help of this parametrization and considering the gauge and CP invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ as:

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1(\phi_1^\dagger \phi_1 - v^2/2)^2 + c_2(\phi_2^\dagger \phi_2)^2 \\ &+ c_3[(\phi_1^\dagger \phi_1 - v^2/2) + \phi_2^\dagger \phi_2]^2 + c_4[(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1)] \\ &+ c_5[Re(\phi_1^\dagger \phi_2)]^2 + c_6[Im(\phi_1^\dagger \phi_2)]^2 + c_7 , \end{aligned} \quad (7)$$

the SM particles and new particles beyond can be collected in the first and second doublets respectively. Here H^0 is the SM Higgs boson and H_1 (H_2) are the new neutral Higgs particles. Since there is no mixing of neutral Higgs bosons at tree level for this choice of Higgs doublets, H_1 (H_2) is the usual scalar (pseudoscalar) h^0 (A^0).

In the Yukawa interaction eq. (4), the part which is responsible for the FCNC at tree level reads as

$$\mathcal{L}_{Y,FC} = \xi_{ij}^E \bar{l}_{iL} \phi_2 E_{jR} + h.c. . \quad (8)$$

Notice that, in the following we will replace ξ_{ij}^E by $\xi_{N,ij}^E$ to emphasize that the couplings are related to the neutral interactions. The Yukawa matrices $\xi_{N,ij}^E$ have in general complex entries and they are free parameters which should be fixed by using the various experimental results.

The effective interaction for the anomalous magnetic moment of the lepton is defined as

$$\mathcal{L}_{AMM} = a_l \frac{e}{4m_l} \bar{l} \sigma_{\mu\nu} l F^{\mu\nu} , \quad (9)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and " a_l " is the AMM of the lepton " l ", ($l = e, \mu, \tau$). This interaction can be induced by the neutral Higgs bosons h^0 and A^0 at loop level in the model III, beyond the SM. As mentioned we do not take charged FC interaction in the leptonic sector due to the small couplings for $\mu - \nu_l$ interactions.

In Fig. 1, we present the 1-loop diagrams due to neutral Higgs particles. Since, the self energy $\Sigma(p)$ (diagrams a, b in Fig. 1) vanishes when the l -lepton is on-shell, in the on-shell renormalization scheme, only the vertex diagram c in Fig. 1 contributes to the calculation of

the AMM of the lepton l . The most general Lorentz-invariant form of the coupling of a charged lepton to a photon of four-momentum q_ν can be written as

$$\begin{aligned}\Gamma_\mu &= G_1(q^2) \gamma_\mu + G_2(q^2) \sigma_{\mu\nu} q^\nu \\ &+ G_3(q^2) \sigma_{\mu\nu} \gamma_5 q^\nu\end{aligned}\quad (10)$$

where q_ν is the photon 4-vector and the q^2 dependent form factors $G_1(q^2)$, $G_2(q^2)$ and $G_3(q^2)$ are proportional to the charge, AMM and EDM of the l -lepton respectively. Using the definition of AMM of the lepton l (eq. (9)), $\Delta_{New} a_\mu$ is extracted as

$$\Delta_{New} a_\mu = a_\mu^{(1)} + \int_0^1 a_\mu^{(2)}(x) dx, \quad (11)$$

where $a_\mu^{(1)}$ ($\int_0^1 a_\mu^{(2)}(x) dx$) is the contribution coming from the internal τ (μ) lepton. The functions $a_\mu^{(1)}$ and $a_\mu^{(2)}$ are given by

$$\begin{aligned}a_\mu^{(1)} &= \frac{G_F}{\sqrt{2}} \frac{Q_\tau}{64 \pi^2} \left\{ \frac{1}{2} ((\bar{\xi}_{N,\mu\tau}^{E*})^2 + (\bar{\xi}_{N,\tau\mu}^E)^2) (F_1(y_{h^0}) - F_1(y_{A^0})) \right. \\ &+ \left. \frac{1}{3} |\bar{\xi}_{N,\tau\mu}^E|^2 \frac{m_\mu}{m_\tau} (G_1(y_{h^0}) + G_1(y_{A^0})) \right\},\end{aligned}\quad (12)$$

and

$$\begin{aligned}a_\mu^{(2)}(x) &= \frac{G_F}{\sqrt{2}} \frac{Q_\mu}{64 \pi^2} (x-1)^2 \left\{ \frac{(\bar{\xi}_{N,\mu\mu}^{E*})^2 + (\bar{\xi}_{N,\mu\mu}^E)^2 + 2 |\bar{\xi}_{N,\mu\mu}^E|^2 x}{1 + (r_{h^0} - 2)x + x^2} \right. \\ &- \left. \frac{(\bar{\xi}_{N,\mu\mu}^{E*})^2 + (\bar{\xi}_{N,\mu\mu}^E)^2 - 2 |\bar{\xi}_{N,\mu\mu}^E|^2 x}{1 + (r_{A^0} - 2)x + x^2} \right\},\end{aligned}\quad (13)$$

where $F_1(w)$ and $G_1(w)$ are

$$\begin{aligned}F_1(w) &= \frac{w(3 - 4w + w^2 + 2 \ln w)}{(-1 + w)^3}, \\ G_1(w) &= \frac{w(2 + 3w - 6w^2 + w^3 + 6w \ln w)}{(-1 + w)^4}\end{aligned}\quad (14)$$

Here $y_H = \frac{m_\tau^2}{m_H^2}$ and $r_H = \frac{m_\mu^2}{m_H^2}$, Q_τ and Q_μ are the charges of τ and μ leptons respectively. In eqs. (12) and (13) $\bar{\xi}_{N,ij}^E$ is defined as $\xi_{N,ij}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \bar{\xi}_{N,ij}^E$. In eq. (11) we take into account internal τ and μ -lepton contributions since, the Yukawa couplings $\bar{\xi}_{N,ij}^E$ i (or j) = e are negligible (see Discussion part). Notice that we make our calculations for arbitrary q^2 and take $q^2 = 0$ at the end.

In our analysis we take the couplings $\bar{\xi}_{N,\tau\mu}^E$ and $\bar{\xi}_{N,\mu\mu}^E$ complex in general and use the parametrization

$$\bar{\xi}_{N,l'l'}^E = |\bar{\xi}_{N,l'l'}^E| e^{i\theta_{l'l'}}. \quad (15)$$

The Yukawa factors in eqs. (12) and (13) can be written as

$$((\bar{\xi}_{N,l'l'}^E)^2 + (\bar{\xi}_{N,l'l}^E)^2) = 2 \cos 2\theta_{ll'} |\bar{\xi}_{N,l'l}^E|^2 \quad (16)$$

where $l, l' = \mu, \tau$. Here $\theta_{ll'}$ are CP violating parameters which lead to the existence of the lepton electric dipole moment.

3 Discussion

The new physics contribution to the AMM of the lepton is controlled by the Yukawa couplings $\bar{\xi}_{N,ij}^E, i, j = e, \mu, \tau$ in the model III. These couplings can be complex in general and they are free parameters of the model under consideration. The relevant interaction (see eq. (9)) can be created by the mediation of the neutral Higgs bosons h^0 and A^0 beyond the SM, with internal leptons e, μ, τ (Fig. 1). However, in our predictions, we assume that the Yukawa couplings $\bar{\xi}_{N,\tau e}^E$ and $\bar{\xi}_{N,\mu\mu}^E$ are small compared to $\bar{\xi}_{N,\tau\mu}^E$ since their strength is proportional to the masses of the leptons denoted by their indices, similar to the Cheng-Sher scenerio [26]. Notice that, we also assume $\bar{\xi}_{N,ij}^E$ as symmetric with respect to the indices i and j . Therefore, the number of free Yukawa couplings is reduced by two and one more coupling, namely $\bar{\xi}_{N,\tau\mu}^E$ still exists as a free parameter. This parameter can be restricted by using the experimental result of the μ EDM [27]

$$d_\mu < 10.34 \times 10^{-19} e - cm . \quad (17)$$

at 95% CL limit and the corresponding theoretical result for the EDM of the muon in the model III (see [25] for details). Since a non-zero EDM can be obtained in the case of complex couplings, there exist a CP violating parameter $\theta_{\tau\mu}$ coming from the parametrization eq. (15). Using the experimental restriction in eq.(17), the upper limit of the coupling $\bar{\xi}_{N,\tau\mu}^E$ is predicted at the order of the magnitude of $10^3 GeV$.

The other possibility to get a constraint for the upper limit of $\bar{\xi}_{N,\tau\mu}^E$ is to use the experimental result of the muon AMM. In this work, we study the new physics effects on the muon AMM and predict a more stringent bound for the coupling $\bar{\xi}_{N,\tau\mu}^E$, with the assumption that the new physics effects are of the order of the experimental uncertainty of the muon AMM measurement. We also check the effect of the coupling $\bar{\xi}_{N,\mu\mu}^E$ on AMM of the muon and observe that AMM has a weak sensitivity on this coupling. This insensitivity is due to the suppression coming from the factors r_{h^0} and r_{A^0} in the denominator of eq. (13). Therefore, we can take $\bar{\xi}_{N,\tau\mu}^E$ as the only free parameter.

Fig. 2 shows the $|\bar{\xi}_{N,\tau\mu}^E|$ dependence of $\Delta_{New}a_\mu$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h^0} = 85\text{ GeV}$ and $m_{A^0} = 95\text{ GeV}$. Here, $\Delta_{New}a_\mu$ is of the order of magnitude 10^{-9} , increases with increasing value of the coupling $|\bar{\xi}_{N,\tau\mu}^E|$ and exceeds the experimental uncertainty, namely 10^{-9} . This forces us to restrict the coupling $|\bar{\xi}_{N,\tau\mu}^E|$ as $|\bar{\xi}_{N,\tau\mu}^E| < 30 \pm 5\text{ GeV}$ for intermediate values of $\sin\theta_{\tau\mu}$, $0.4 \leq \sin\theta_{\tau\mu} \leq 0.6$. This is a much better upper limit compared to the one obtained using the experimental result of the μ EDM.

In Fig. 3, we show the $\sin\theta_{\tau\mu}$ dependence of $\Delta_{New}a_\mu$ for $\bar{\xi}_{N,\tau\mu}^E = 30\text{ GeV}$, $m_{h^0} = 85\text{ GeV}$ and $m_{A^0} = 95\text{ GeV}$. Increasing values of $\sin\theta_{\tau\mu}$ cause $\Delta_{New}a_\mu$ to decrease and to lie within the experimental uncertainty.

In Fig. 4, we present m_{h^0} dependence of $\Delta_{New}a_\mu$ for $\bar{\xi}_{N,\tau\mu}^E = 30\text{ GeV}$, $\sin\theta_{\tau\mu} = 0.5$, and $m_{A^0} = 95\text{ GeV}$. The upper limit of $\Delta_{New}a_\mu$ decreases with increasing values of m_{h^0} .

For completeness, we also show the $|\xi_{N,\mu\mu}^E|$ dependence of $\Delta_{New}a_\mu$ when the internal μ -lepton contribution is taken into account. In this figure, it is observed that $\Delta_{New}a_\mu$ is only weakly sensitive to $|\xi_{N,\mu\mu}^E|$ for $|\xi_{N,\mu\mu}^E| < 0.1\text{ GeV}$ and therefore the internal μ -lepton contribution can be safely neglected, for these values.

In this work, we choose the type III 2HDM of [1] for the physics beyond the SM and assume that only FCNC interactions exist at tree level, with complex Yukawa couplings. We predict an upper limit for the coupling $|\bar{\xi}_{N,\tau\mu}^E|$ for the intermediate values of the imaginary part, by assuming that the new physics effects are of the order of the experimental uncertainty of muon AMM, namely 10^{-9} , and see that this leads to a much better upper limit, $\sim 30\text{ GeV}$, compared to the one obtained by using the experimental result of the μ EDM, $\sim 10^3\text{ GeV}$. In the calculations, we studied the internal μ lepton contributions as well. However, we observe that they give a negligible contribution to the AMM of muon. Furthermore, we neglect the e lepton contribution. With more accurate future measurements of the AMM, it should be possible to constrain the parameters of the Two Higgs Doublet Model more stringently.

4 Acknowledgement

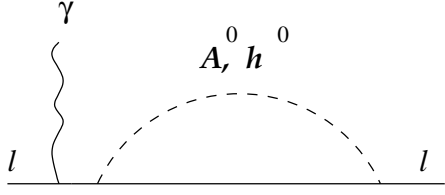
This work has been supported by the Turkish Academy of Sciences, in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8).

References

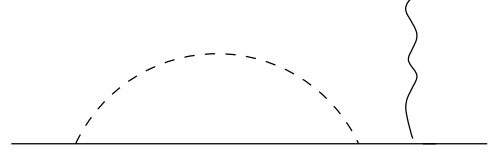
- [1] S. Nie, M. Sher, *Phys. Rev.* **D58**, 097701 (1998);

- [2] J. L. Lopez, D.V. Nanopoulos and X. Wang, *Phys. Rev.* **D49**, 366 (1994); U. Chattopadhyay and P. Nath, *Phys. Rev.* **D53**, 1648 (1996); T. Moroi, *Phys. Rev.* **D53**, 6565 (1996); Erratum-ibid, **D56**, 4424 (1997); M. Carena, G. F. Giudice and C. E. M. Wagner, *Phys. Lett.* **B390**, 234 (1997); E. Gabrielli and U. Sarid, *Phys. Rev. Lett.* **79**, 4752 (1997); K.T. Mahanthappa and S. Oh, *Phys. Rev.* **62**, 015012 (2000); J. L. Feng and T. Moroi, *Phys. Rev.* **D61**, 095004 (2000); T. Goto, Y. Okada and Y. Shimizu, hep-ph/9908499; T. Blazek, hep-ph/9912460; U. Chattopadhyay, D. K. Ghosh and S. Roy, *Phys. Rev.* **D62**, 115001 (2000).
- [3] H. N. Brown et.al, Muon g-2 Collaboaration, *Phys.Rev.Lett.* **86** (2001) 2227.
- [4] M. Deile, et. al., Muon g-2 Collaboaration, hep-ex/0211034 (2002).
- [5] M. Davier et. al., hep-ph/0208177, and references therein
- [6] Akhmetshin et.al. , *Phys. Lett.* **B527** (2002) 161.
- [7] K. Lane, hep-ph/0102131.
- [8] L. Everett, G. L. Kane, S. Rigolin and L. T. Wang, *Phys. Rev. Lett.* **86**, (2001) 3484.
- [9] J. L. Feng and K. T. Matchev, *Phys. Rev. Lett.* **86**, (2001) 3480.
- [10] E. A. Baltz and P. Gondolo, *Phys. Rev. Lett.* **86**, (2001) 5004.
- [11] D. Chakraverty, D. Choudbury and A. Datta, *Phys. Lett.* **B506**, (2001) 103.
- [12] T. Huang, Z. H. Lin, L.Y. Shan and X. Zhang, *Phys.Rev.* **D64** (2001) 071301.
- [13] D. Choudhury, B. Mukhopadhyaya and S. Rakshit, *Phys. Lett.* **B507**, (2001) 219.
- [14] S. Komine, T. Moroi and M. Yamaguchi, *Phys. Lett.* **B506**, (2001) 93.
- [15] S. N. Gninenko and N. V. Krasnikov, *Phys. Lett.* **B513** (2001) 119
- [16] K. Cheung, *Phys. Rev.* **D64** (2001) 033001.
- [17] P. Das, S. K. Rai and S. Raychaudhuri, hep-ph/0102242.
- [18] T. W. Kephart and H. Pas, hep-ph/0102243.
- [19] Z. Xiong and J. M. Yang, *Phys.Lett.* **B508** (2001) 295

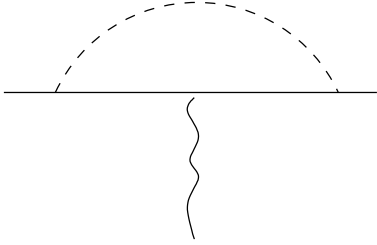
- [20] A. Dedes and H. H. Haber, *JHEP* **0105** (2001) 006
- [21] R. A. Diaz, R. Martinez and J-A. Rodriguez, *Phys.Rev.* **D64** (2001) 033004
- [22] S. K. Kang and K. Y. Lee, *Phys.Lett.* **B521** (2001) 61
- [23] S. C. Park and H. S. Song, *Phys. Lett.* **B506**, (2001) 99.
- [24] C. A. de S. Pires and P. S. Rodrigues, *Phys.Rev. D* **64** (2001) 117701.
- [25] E. Iltan, *Phys. Rev.* **D64**, (2001) 013013.
- [26] T. P. Cheng and M. Sher, *Phys. Rev.* **D35**, (1987) 3383.
- [27] J. Bailey et. al., *Journ Phys.* **G4**, (1978) 345.



(a)



(b)



(c)

Figure 1: One loop diagrams contributing to AMM of l -lepton due to the neutral Higgs bosons h^0 and A^0 in the 2HDM. Wavy (dashed) line represents the electromagnetic field (h^0 or A^0 fields).

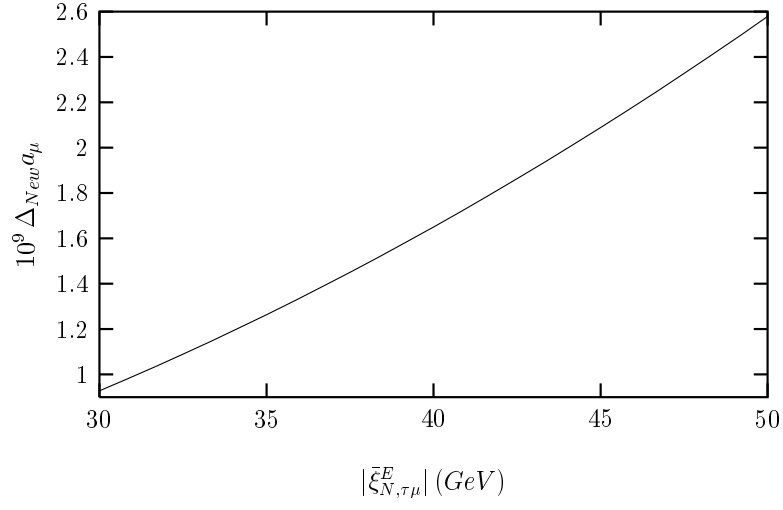


Figure 2: $\Delta_{N_{ew}} a_\mu$ as a function of $|\bar{\xi}_{N,\tau\mu}^E|$ for $\sin\theta_{\tau\mu} = 0.5$, $m_{h^0} = 85 \text{ GeV}$ and $m_{A^0} = 95 \text{ GeV}$.

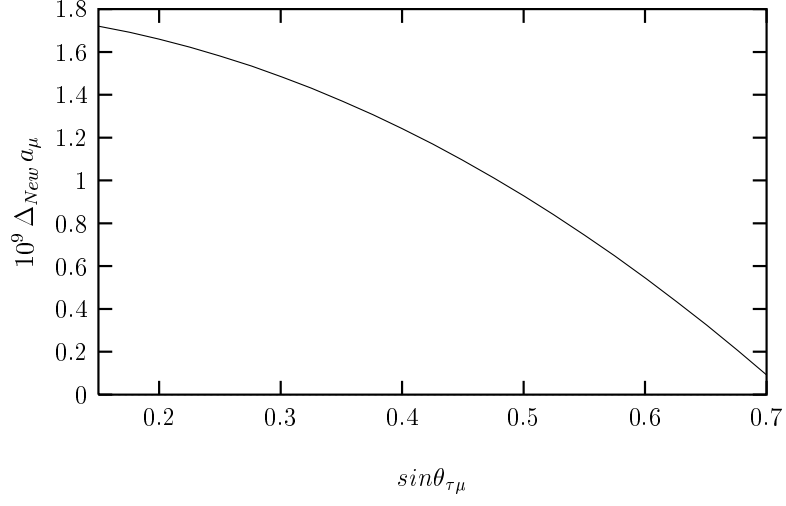


Figure 3: $\Delta_{New} a_\mu$ as a function of $\sin \theta_{\tau\mu}$ for $|\bar{\xi}_{N,\tau\mu}^E| = 30 \text{ GeV}$, $m_{h^0} = 85 \text{ GeV}$ and $m_{A^0} = 95 \text{ GeV}$.

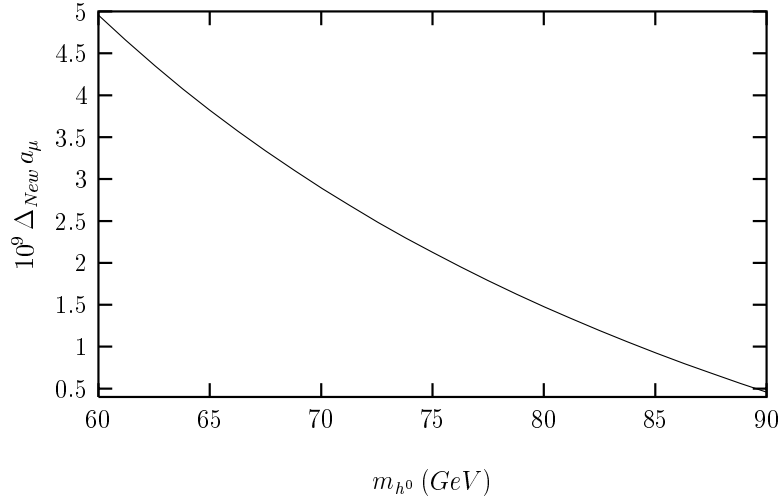


Figure 4: $\Delta_{New} a_\mu$ as a function of m_{h^0} for $|\bar{\xi}_{N,\tau\mu}^E| = 30 \text{ GeV}$, $m_{A^0} = 95 \text{ GeV}$ and $\sin \theta_{\tau\mu} = 0.5$.

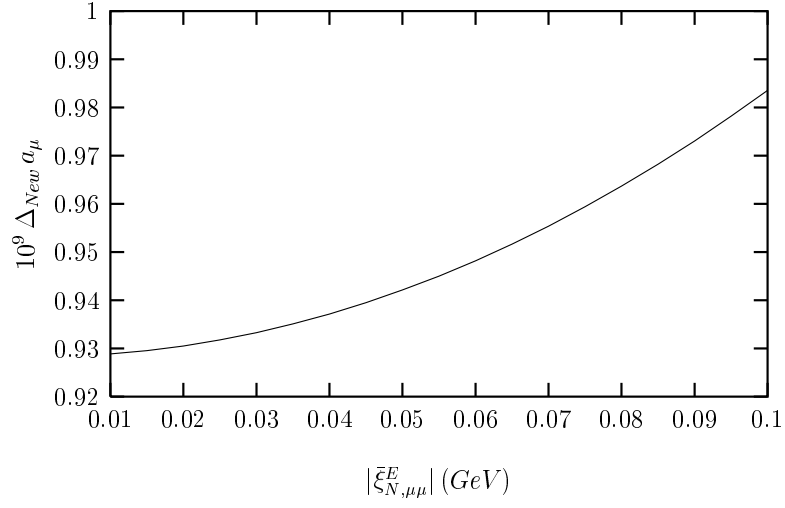


Figure 5: $\Delta_{New} a_\mu$ as a function of $|\xi_{N,\mu\mu}^E|$ for $|\bar{\xi}_{N,\tau\mu}^E| = 30 \text{ GeV}$, $\sin \theta_{\tau\mu} = 0.5$, $\sin \theta_{\mu\mu} = 0.5$, $m_{h^0} = 85 \text{ GeV}$ and $m_{A^0} = 95 \text{ GeV}$.